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An evaluation of alternative forecasting techniques in a replacement market subject to independent demand.

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AN EVALUATION OF ALTERNATIVE FORECASTING TECHNIQUES IN A REPLACEMENT MARKET SUBJECT TO INDEPENDENT DEMAND

by

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ABSTRACT

The investigation reported on in this thesis concerned the evaluation of alternative forecasting models given a replacement market subject to independent demand. Initially, basic forecasting techniques were examined. Extending these basic models, two forecast monitoring systems were added to increase model responsiveness to change in the underlying demand process. The six models examined were the six-month moving average, six-month weighted moving average, single exponential smoothing, single smoothing with a linear trend, double smoothing, and single smoothing with adaptive control.

With this foundation, criteria for the evaluation of model goodness-of-fit were developed. The three criteria used were the Theil's U-Statistic, average absolute percent error, and analysis of error distributions. Next, the model parameters of the nonadaptive exponential smoothing models were optimized. Finally, all selected forecasting techniques were compared on the basis of the evaluation criteria. As a result of this analysis, a six month weighted moving average model was selected as the best forecasting technique for the industry examined, ranking first on every evaluation criterion.

CHAPTER 1

INTRODUCTION

This thesis studies the feasibility for the development of a system to evaluate alternative forecasting methods for organizations confronting a replacment market subject to independent demand. The basic objectives of the thesis can be stated as follows:

Examine basic forecasting techniques

Establish criteria for evaluating forecasting models

Present analyses succinctly

Basic prediction models are examined to provide a background in forecasting techniques. The models were selected on the basis of their low degree of mathematical complexity and limited requirements for historical data. Simpler models are necessary for organizations with limited forecasting experience. To sucessfully apply and evaluate a forecasting system, it is necessary to understand the system being examined. Forecasting systems which require many periods of historical data are unacceptable for two reasons. First, a large number of forecasted items may make the total data requirements of the system excessive. Second, many forecasted items in such organizations have limited demand history and relatively short product life.

Criteria for the evaluation of forecasting methods are examined along with their theoretical bases. These criteria provide yardsticks for the comparison of alternative forecasting techniques.

The basic thesis work performed consisted of reviewing existing systems, analyzing several aspects of the organizational environment, and developing a procedure to meet the stated objectives.

The procedure which resulted can be viewed as a three step process. First, alternative forecasting methods are selected. Second, model parameters are optimized. Finally, the systems are compared using the selected evaluation criteria.

CHAPTER 2

PROBLEM DEFINITION

The predicition of future demand is a key factor in establishing proper inventory policy and production planning. Therefore, it is essential that an organization use the best techniques available.

Many business firms are unable to justify the services of a full-time forecaster because of their limited size. Others, produce such a large variety of products that the selection of the best technique for each and every item would be an overwhelming task. These factors combine to make the selection of the "best" forecasting technique nearly impossible.

This thesis work outlines a procedure for the testing and evaluation of several alternative forecasting methods given a replacement market subject to independent demand. In order to achieve the goal of finding the best forecasting technique, several assumptions have been followed.

First, parsimony is an important consideration in the choice of a forecasting technique. In other words, simple models are better than complex ones. Complex models are inherently difficult to apply and generally more dificult to explain to others. Acceptance of a forecasting method by those who must use it, namely production and inventory managers, is a major factor in determining the sucess of a model. Therefore, one criterion for the best model is the method be understandable to those who must use it.

Second, there is no best forecasting technique, only a most appropriate prediction method. Forecasting is a time varying process, and over time what was the best analysis technique may not continue to be best. Therefore, it is essential that any forecasting method provide for the detection of changes in the underlying demand process. The models examined in this analysis provide for this through tracking signals or adaptive response of their model parameters. These monitoring systems permit the forecasting system to detect when its forecasts are no longer appropriate.

Third, the evaluation of alternative methods should not be made on a single criterion but from several criteria. No one statistic completely evaluates the goodness-of-fit of a forecasting model. These statistics must be examined separately and combined by the analyst to determine the best forecasting technique. Finally, even though different models may be best for different items, it is advantageous to apply a single model for all items when the number is very large. In other words, the best model is the model which functions best across the greatest variety of items.

In summary, the problem is to develop a system to identify the best forecasting model for a collection of manufactured items where the number of different products is large and the forecasting capabilities of the organization are limited.

CHAPTER 3

BACKGROUND

The forecasting techniques examined in this analysis fall into two broad catigories, constant and trend models. Some of these models have been enhanced by the addition of tracking systems. One tracking system uses the cummulative sum of forecast errors, while the other is a method of automatically adjusting model parameters as the underlying process changes.

CONSTANT MODELS

Constant models are used to predict future events when the underlying process is assumed to be constant. If a trend is present, constant models will lag the series yeilding consistantly biased results. However, when the assumption of constant demand is acceptable, these models provide the most parsimonious description of the demand process.

Moving Average Model

One of the earliest methods of forecasting future events in a time series was to compute the average of the most recent entries in the series. The most recent entries, most often over the past six, 12, or 13 periods, are assumed to be random samples from a normal distribution of demand. The mean of this distribution would then represent the best predictor of future entries. Over time, older information is removed from the computational model, while new information, which is considered more meanful, is added. Hence the name moving average was derived. Mathematically, the basic moving average model is as follows:

$$F(t) = \frac{D(t-1) + D(t-2) + \ldots + D(t-n)}{n}$$

where D(i) = demand at time i
n = number of periods in the moving average

This work examines a six-month moving average model which is designated MAV.

Weighted Moving Average Model

A natural extention of the moving average model is the weighted moving average. The weighted moving average model places more importantance or weight on more recent entries in the series. The basic form of the model is as follows:

$$F(t) = A \cdot D(t-1) + B \cdot D(t-2) + \ldots + Z \cdot D(t-n)$$

$$\text{where} \quad A + B + \ldots + Z = 1$$

$$\text{and} \quad A > B > \ldots > Z > 0$$

The predicted value, $F(t)$, is the sum of the n most recent entries discounted by proportional weighting factors $(A, B, \ldots, Z)$. The underlying assumption of the model is that newer information is more valuable than old information and therefore should have more predicting power in the forecast model. This work examines a six-month weighted moving average which is designated WMA.

Exponential Smoothing Model

The exponential smoothing model is an extention of the weighted moving average model. Exponential smoothing weights all past entries in an exponentially decaying manner. The basic form of the model is as follows:

$$F(t) = \text{Alpha} \cdot D(t-1) + (1 - \text{Alpha}) \cdot F(t-1)$$

where $0.0 < Alpha < 0.5$

The previous forecast, F(t-1), incorporates all past demands with more recent entries comprising the most significant portion of this forecast. The previous forecast is then discounted by a factor of (1-Alpha), effectively reducing the importance of older information. The most recent demand, D(t-1), is multiplied by Alpha and added to the discounted forecast to generate the new forecast. Since the previous forecast is comprised of all past demands, the most recent demand becomes the most significant portion of the present forecast. The cycle continues with older information becoming less and less important.

This model has several advantages over the moving average and weighted moving average models and is the most widely used of the constant series models. The model can be made to react more quickly to change by increasing the value of Alpha, thereby increasing the importance of more recent entries in the series. Exponential smoothing also has the advantage of much lower data requirements than other constant models. It requires only the most recent entry in the series and the previous forecast to generate a new forecast. This is extremely advantagious where a large number of inventory items must be predicted.

TREND MODELS

Trend models, unlike constant models, explicitly provide for trends in the process being analyzed. This work examines two such models: double exponential smoothing and single smoothing with a linear trend. Both of these models have the advantage of low data requirements which is an important consideration when a large number of inventory items must be forecast.

Double Exponential Smoothing

Double exponential smoothing [5] is an extention of single smoothing which uses the current forecast error as the principle basis from which the model parameters are updated. The model parameters include level, A(t), and slope, B(t), terms which define a line with a value of A(t) and a slope of B(t) at time t. By providing for slope in the fitted model, double exponential smoothing permits trend in the underlying process to be explored. The basic form of the model is as follows:

$$A(t) = D(t) + (1-\text{Alpha}) \cdot E(t)$$

$$B(t) = B(t-1) + (\text{Alpha}) \cdot E(t)$$

where $A(t)$ and $B(t)$ are model parameters and $D(t)$ and $E(t)$ are the most recent demand and forecast error respectively with $E(t) = F(t) - D(t)$

The one period forecast is given by:

$$F(t+1) = A(t) + B(t)$$

and in general the r-period-ahead forecast is given by:

$$F(t+r) = A(t) + B(t) \cdot r$$

Single Smoothing with a Linear Trend

Single smoothing with a linear trend [9], similar to double smothing, adjusts the level, $A(t)$, and slope, $B(t)$, directly. These parameters are updated using the following relationships:

$$A(t) = \text{Alpha} \cdot D(t-1) + (1-\text{Alpha}) \cdot F(t-1)$$

$$B(t) = \text{Alpha} \cdot (A(t)-A(t-1)) + (1-\text{Alpha}) \cdot B(t-1)$$

The r-period-ahead forecast is given by:

$$F(t+r) = A(t) + B(t) \cdot r$$

The forecast, F(t+r), is a simple linear combination of the two model parameters, A(t) and B(t). The level, A(t), is simply the single exponential smoothing of past denands. The slope, B(t), is the smoothed value of the change between successive values of the single smoothing forecast.

ADAPTIVE CONTROL MODELS

Adaptive control models provide for the continuous modification of the Alpha parameter in a smoothing model. They increase Alpha when faster model response becomes necessary because of changes in the underlying process and reduce Alpha when actual entries remain consistant with their forecasted values. This work investigates the application of the Trigg and Leach (10) technique for the modification of the Alpha parameter in both the single and double smoothing models. A second method of adaptive control provides a tracking signal which, when triggered, signals that the forecasting model is out-of-control. This cummulative sum method of monitoring has been applied to the single, double, and single smoothing with a linear trend models.

Trigg and Leach Adaptive Response Rate

The Trigg and Leach technique requires that two smoothed error values be calculated, smoothed average error and smoothed absolute error. These values are updated using the following relationships:

$$SE(t) = Gamma \cdot E(t) + (1-Gamma) \cdot SE(t-1)$$

$$AE(t) = Gamma \cdot |E(t)| + (1-Gamma) \cdot AE(t-1)$$

where Gamma = error smoothing parameter
E(t) = D(t-1) - F(t-1) = current forecast error
SE(t) = smoothed average error
AE(t) = smoothed absolute error

The two error values, SE(t) and AE(t), are combined to yield an additional measure of error called a tracking signal using the relationship:

$$TS(t) = \frac{SE(t)}{AE(t)}$$

The tracking signal is then used to compute the smoothing parameter, Alpha, for the next period using the relationship:

$$\text{Alpha} = \left| TS(t) \right|$$

Since the smoothed error can never excede absolute error, the value of the tracking signal will always fall between -1 and 1. In the case of double exponential smoothing the modified Alpha parameter is used only to estimate slope, B(t). If it were use to estimate both level, A(t), and slope, B(t), the system would be overly reactive to random error. Such a system would oscillate about the true underlying process and, therefore, be a poor prediction tool. Two of these adaptive control models were examined, single smoothing with adaptive control (SAC) and double smoothing with adaptive control (DAC).

Cummulative Sum Tracking Signal

Brown [9] developed a tracking signal which can also be used with smoothing models. The tracking signal is intialized at zero and thereafter follows the generated forecast error. If the model is biased, the signal will move away from zero and eventually violate the tracking signal limits. Mathematically, the tracking signal is computed from:

$$TS(t) = \frac{Y(t)}{MAD(t)}$$

where $Y(t) = \sum_{i=c}^{n} E(i)$ = cumulative sum of forecast errors since the model parameters were intilaized at period c

$E(t)$ = error at time t

$MAD(t)$ = mean absolute deviation of forecast errors at time t

The mean absolute deviation, MAD(t), is a smoothed average of the absolute forecast errors and is calculated using the relationship:

$$MAD(t) = Alpha \cdot |E(t)| + (1-Alpha) \cdot MAD(t-1)$$

where Alpha = smoothing parameter used in the smoothing model

The confidence interval for the tracking signal is expressed in terms of standard deviations of tracking signal variability (eg., two standard deviations would yield a 95% confidence interval). The standard deviation of the tracking signal can be derived from:

$$\text{Sigma}_{TS} = 0.884 \sqrt{\frac{1 + \text{Beta}}{1 - \text{Beta}}}^{\,2n}$$

where Beta = 1- Alpha
n = 1 for single smoothing
n = 2 for double smoothing

If the tracking signal should fall outside the range:

$$-(\text{Conf}) \cdot \text{Sigma}_{TS} < TS(t) < (\text{Conf}) \cdot \text{Sigma}_{TS}$$

where Conf = number of standard devations of tracking signal variability which fall in the desired confidence interval

the forecasts are assumed to no longer be valid, the model parameters must be reset, and the tracking signal reintialized. This tracking system was used with single (SET), double (DET), and single smoothing with a linear trend (SLT).

CHAPTER 4

EVALUATION CRITERIA

The analysis of alternative forecasting methods was conducted using a comparison of two broad measures of forecasting reliability. These evaluation criteria were the average absolute percent deviation and the Theil's U-statistic. Both provide the analyst with a measure of the goodness-of-fit of forecasting models for the time series being analyzed. Comparitively, neither method is superior, but each provides a basis for the comparison of forecasting techniques.

Time series contain random variation which cannot be anticipated and which should not affect the forecasting model. If the predictions are affected by random variation, the reliability of the forecasts will be reduced as the model attempts to follow random error.

Forecasted series should represent the underlying process with the random variation removed. The two criteria for evaluating the ability of a model to remove this random fluctuation are discussed below. Finally, the distribution of the forecast errors were analyzed to detect model bias and the variability of the error terms.

Average Absolute Percent Error

The average absolute percent deviation (AAPD) is a direct method for the comparison of the difference between the forecasted series and the actual series. Because it provides a standardized measure of model fit, this method can be applied across many series of varying magnitude. Mathemetically, the average absolute percent deviation in computed as follows:

$$AAPE = \frac{\sum_{i=1}^{n} \frac{F(i) - D(i)}{F(i)}}{n}$$

where F(i)= Forecast at time i
D(i)= Demand at time i
n = Number of periods

The absolute error at time i is the absolute difference between the forecasted value, F(i), and the actual demand, D(i). This error is standardized by dividing by the forecasted value to obtain the absolute percent deviation of the forecast. Because the forecasted series is assumed to represent the true process with the effects of random variation removed, the predicted value is used in the standardization procedure. The average value of the absolute percent deviation is then used as an indicator of the goodness-of-fit of the forecasting model, with lower values highlighting better models.

Theis's U-Statistic

Theil's U-statistic [8] provides a measure of model improvement over the assumption that the series is a "random walk." For example, if the series were a random walk, it would be equally probable that the next entry would be above or below its present value. Therefore, the best estimate of the next entry would be the present entry. The Theil's U-statistic is simply a ratio of the forecast errors to the errors derived using the random walk assumption. Mathematically, the Theil's U-statistic is computed as follows:

$$U = \sqrt{\frac{MSE(f)}{MSE(r)}}$$

where MSE(f) = mean square forecast error as a percent of demand
MSE(r) = mean square error as a percent of demand using the random walk assumption

The mean square error values are computed from:

$$\mathrm{MSE}(f) = \frac{\displaystyle\sum_{i=1}^{n} \frac{(F(i) - D(i))^2}{D(i)}}{n}$$

$$\mathrm{MSE}(r) = \frac{\displaystyle\sum_{i=1}^{n} \frac{(D(i-1) - D(i))^2}{D(i)}}{n}$$

The MSE(f) and MSE(r) values are standardized using the demand, D(t), at each period. In this manner, both values are derived from a common basis. The square root of the ratio of these values is the Theil's U-statistic. Values greater than 1.0 indicate that the random walk assumption provides a better predictor of future entries than the forecasting model. Between 1.0 and 0.0, the Theil's U-statistic provides a relative comparison of goodness-of-fit between models. A smaller Theil's U-statistic denotes improved forecasting accuracy, with a value of 0.0 indicating a perfect model fit since all forecast errors would be zero.

Analysis of Error Distribution

Forecast errors are assumed to be the random variation in the process being observed. They are assumed to be randomly distributed with a mean of zero. If the mean is significantly different from zero, the model is said to be a bias predictor of future events. Bias models result in forecasts which are consistantly too high or too low. This assumption is tested using a t-statistic [7] where the null hypothesis is that the mean is equal to zero. A second measure, standard deviation, is used to evaluate the variability of the forecast error distribution. Small standard deviation denotes an error distribution with low variability. Simply, the error terms are less volitile; consequently, they are not subject to extreme values. Therefore, it is very desirable to find a forecasting model which generates unbias errors which have low variability.

CHAPTER 5

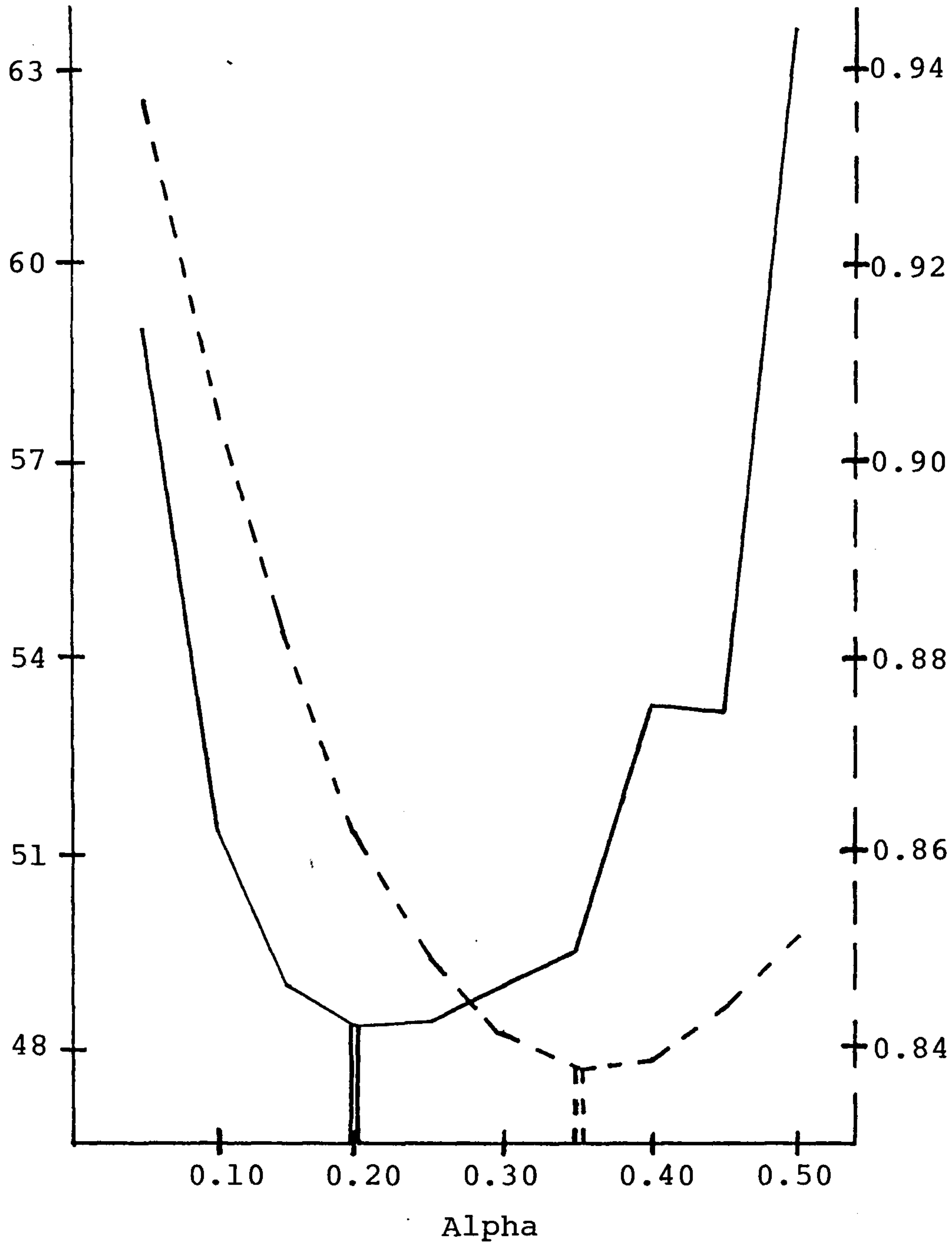
ANALYSIS

Fifteen demand series consisting of thirty-three periods were forecasted using each of the seven methods presented in Chapter 3. The data was obtained from the sales records of a medium size manufacturing firm. The analysis of the seven alternative forecasting techniques selected was a two-step process. First, the model parameters of each of the three nonadaptive exponential smoothing models and the weighted moving average model were optimized. Second, a comparative analysis of all seven models was made using selected evaluation criteria.

OPTIMIZATION OF MODEL PARAMETERS

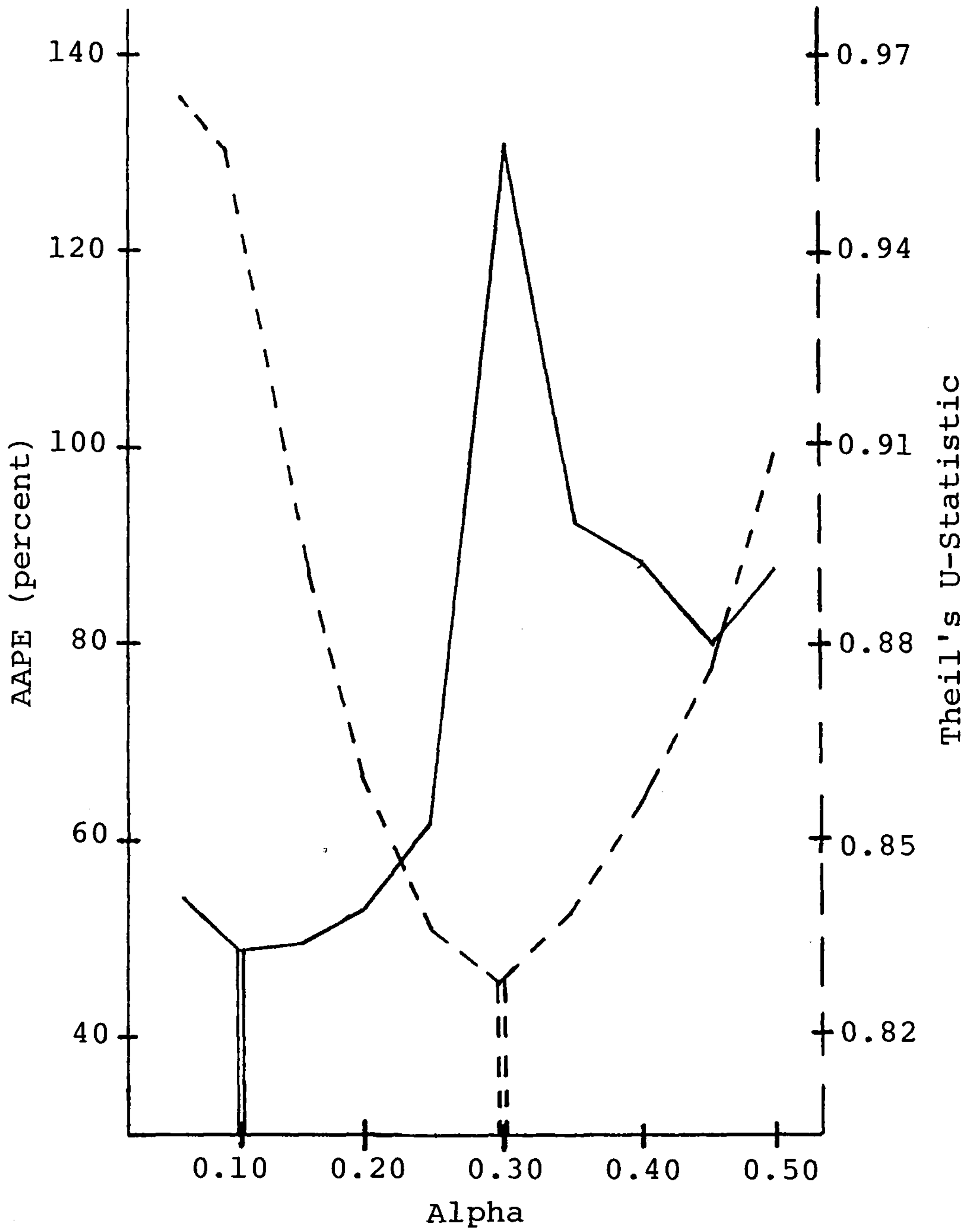
The Alpha optimization procedure required a test of several values of Alpha and the evaluation of model performance on each of the testing criterion. The results of this analysis were plotted and the optimal points were identified on both the AAPE and average Theil's U-statistic curves. These points were assumed to be limiting points for the range of acceptable values of Alpha. Since there is no direct relationship between the two curves, the derivation of a "total cost" type function is impossible. Therefore, the Alpha value was selected out of the acceptable region in order to minimize the loss from the optimal value on both curves. Figures 5-1, 5-2, and 5-3 show the relationship between Alpha and the two evaluation criteria for single smoothing, single smoothing with a linear trend, and double smoothing respectively. The Alpha values selected for the single smoothing,single smoothing with a linear trend, and double smoothing were 0.25, 0.20, and 0.15, respectively.

The model parameters for the weighted moving average model were selected following the examination of several sets of weighting factors. Although regression techniques could optimize model parameters over the sample series, the resulting coefficients would, to a significant degree, be fitted to random error. Therefore, the selection of the model parameters from a group of possible sets is acceptable [9]. These sets represented a wide range of potential wieghtings. Each set of model parameters was evaluated on both the Theil's U-Statistic and the AAPE criteria. As a result, the set of weighting factors selected was 0.30, 0.25, 0.20, 0.15, 0,07, and 0.03 for the most recent to the six-month's prior demand, respectively.



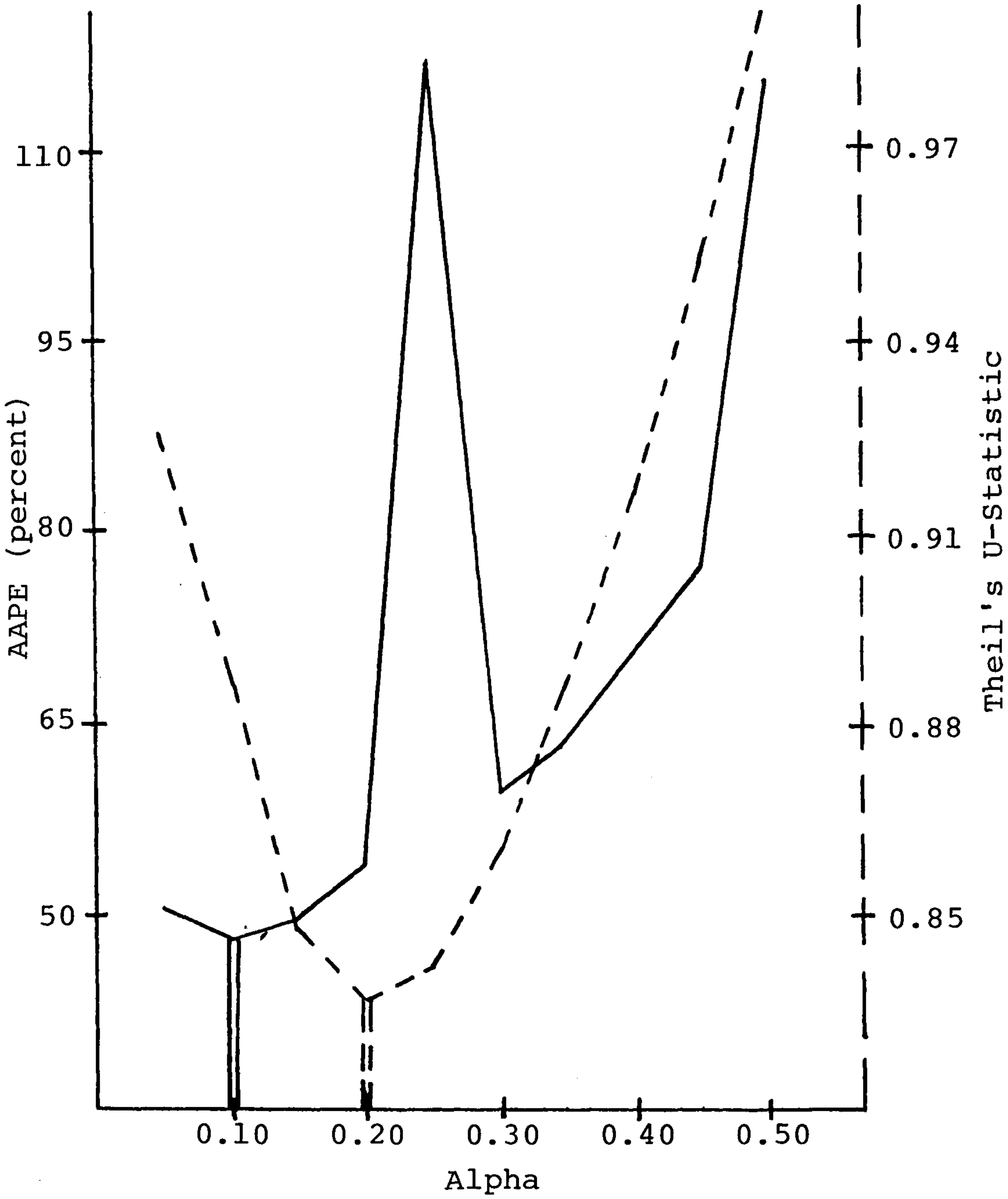
Single Exponential Smoothing
Alpha VS AAPE and Theil's U-Statistic

Figure 5-1



Single Smoothing with a Linear Trend
Alpha VS AAPE and Theil's U-Statistic

Figure 5-2



Double Exponential Smoothing
Alpha VS AAPE and Theil's U-Statistic

Figure 5-3

COMPARATIVE ANALYSIS

The results of the analysis are most easily understood when presented in tabular form. The tables are concise, easy-to-understand, and present a vast amount of information in a succinct manner. The results of the average absolute percent error analysis and the Theil's U-statistic analysis are presented in Tables 5-1 and 5-2 repectively, with combined model rankings presented in Table 5-3. The analysis of error distributions for each forecasting method are summarized in Table 5-4.

Average Absolute Percent Error Analysis

On the basis of the average absolute percent error (AAPE) criterion, the simple six-month weighted moving average (WMA) performed best. The WMA model yielded the lowest average value and had the smallest range of AAPE values. The moving average (MAV), single exponential smoothing (SET), single smoothing with a linear trend (SLT), and double exponential smoothing (DET) were nearly equivalant using the AAPE criterion. However, of these models, the SLT exhibited the narrowest range of AAPE values, which denotes more consistant model performance. The single (SAC) and double smoothing with adaptive

	Moving Average Models		Exponential Smoothing Models			Adaptive Models	
	MAV	WMA	SET	SLT	DET	SAC	DAC
AAPE for Following Month's Forecast	48.80	47.09	48.34	49.95	48.29	54.66	51.95
Range	23.81-98.08	22.91-89.43	24.79-86.04	24.09-78.57	23.98-80.26	28.52-126.25	26.45-106.87
Frequency of Model Ranking							
1st	1	5	3	1	2	0	3
2nd	4	2	2	1	4	2	0
3rd	2	2	1	2	4	3	1
4th	0	1	4	2	2	0	6
5th	3	0	3	4	1	3	1

AAPE Analysis

Table 5-1

control (DAC) were the poorest performers using the AAPE criterion and had the widest range of values. The performance of the SAC model was the weakest of the seven examined using the AAPE criterion.

Theil's U-Statistic Analysis

The MAV, WMA, and SET models were best using the Theil's U-statistic criterion, with the WMA model yielding the narrowest range of Theil's values. The three constant models, MAV, WMA, and SET, yielded the lowest average Thiel's U-statistics indicating the assumption of constant demand is not unreasonable. The remaining models were nearly equal. Of these, the two adaptive control models, SAC and DAC, had the narrowest range of values. In fact, these yielded the smallest of any of the models. The widest range of Theil's U-statistics were generated by the three exponential smoothing models, SET, SLT, and DET, indicating the poorest Theil's performance.

Combined Model Analysis

Considering both evaluation criteria, the WMA model achieved the highest ranking, with eight first place finishes. The MAV and SET models followed with six and

	Moving Average Models		Exponential Smoothing Models			Adaptive Models	
	MAV	WMA	SET	SLT	DET	SAC	DAC
Average Theil's U-Statistic for Following Month's Forecast	0.83	0.81	0.84	0.90	0.88	0.87	0.89
Range	0.39-1.14	0.45-1.03	0.40-1.49	0.41 1.42	0.39-1.44	0.55 1.12	0.55-1.11
Frequency of Model Ranking							
1st	5	3	2	2	1	2	0
2nd	2	4	4	0	1	2	2
3rd	2	1	4	1	3	1	3
4th	1	1	0	2	5	3	3
5th	1	2	3	5	0	2	2

Theil's U-Statistic Analysis

Table 5-2

	Moving Average Models		Exponential Smoothing Models			Adaptive Models	
	MAV	WMA	SET	SLT	DET	SAC	DAC
Frequency of Model Ranking							
1st	6	8	5	3	3	2	3
2nd	6	6	6	1	5	4	2
3rd	4	3	5	3	7	4	4
4th	1	2	4	4	7	3	9
5th	4	2	6	9	1	5	3

Combined Model Analysis

Table 5-3

five, respectively. Combining first and second place finishes, the WMA model again lead with 14 top finishes, trailed by MAV and SET with 11 each. Totaling the top three rankings, the WMA model still placed first with 17, while MAV, SET, and DET followed with 16, 16, and 15, respectively. Overall, the WMA model most consistantly demonstrated the ability to be a top performer.

Error Distribution Analysis

Six of the seven models generated unbiased forecast errors, with SAC being the single exception. The mean sample error of each of the six was not significantly different from zero since it was within the 95% confidence interval of the standard t-test.

The WMA model yielded the smallest standard deviation of error which is consistant with both the AAPE and Theil's analysis. The MAV, SET, DET, and SLT models followed, with the MAV model producing slightly more variable errors. The DAC model yielded highly variable error terms while those of the SAC model were the most extreme.

Model	Mean Error	95% Conf. Intv. of Mean	Standard Deviation of Error
MAV	-10.90	-29.60/7.60	79.91
WMA	-9.18	-25.11/6.75	69.06
SET	-12.22	-30.81/6.38	71.09
SLT	-10.15	-26.95/6.65	74.10
DET	-10.21	-26.84/6.43	71.64
SAC	14.85	9.14/20.56	101.92
DAC	-11.35	-30.62/7.93	88.46

ANALYSIS OF

ERROR DISTRIBUTIONS

TABLE 5-4

CHAPTER 6

CONCLUSIONS

For the industry examined and under the evaluation criteria used, the best overall model investigated was the six-month weighted moving average model, ranking first on every evaluation criterion. The other constant models, the moving average and single exponential smoothing, were the next best models for the sample series examined. The trend and adaptive response models were the poorest predictors of future entries with the former performing better than the later.

Careful examination of the model order will reveal a trend from less responsive models which are better predictors, toward more responsive models which are poorer predictors. This trend highlights a tendency for more responsive models to react to random error in the demand process. The demand series analyzed were very volatile, with the coefficient of variation [1] of demand reaching almost 80% in some cases (Refer to Table 6-1). The coefficient of variation is simply the standard deviation of demand expressed as a percent of the mean value. It could be argued that the high variability was caused by inherent trend in the data, however, trend models were generally poorer predictors than constant models.

Therefore, the high variability of demand would denote a high random error. Moreover, these results strengthen the conclusion that demand can be assumed to be constant.

The more responsive models are adversely affected by high random error. Basically, the models become too responsive and attempt to follow random variation in the process. The resulting forecasts are not free from the effects of chance error because the model bases future entries on this fluctuation. Hence, the models provide relatively poor forecasts compared with their less responsive counterparts.

A second advantage of the weighted moving average model is its ability to totally eliminate older information. In exponential smoothing models, all past information is to some degree contained in the forecast. Any attempt to decrease the reponsiveness of an exponential smoothing system (by reducing Alpha) results in a coresponding increase in the importance of older information. In contrast, the weighted moving average model ignores all but the most recent entries.

Obviously, less responsive models will not always be the best predictors of future events. If the demand series are subject to sudden shifts, contain trend, or have lower variability, more responsive models would be best.

Series	Mean	Standard Deviation	Coefficient of Variability
D1	1265.9	384.9	30.41
D2	421.3	210.1	49.87
D3	3145.2	1220.0	38.79
D4	226.9	146.9	64.74
D5	1651.4	716.2	43.37
D6	1726.8	566.4	32.80
D7	638.9	409.3	64.06
D8	335.5	207.7	61.19
D9	217.1	173.5	79.92
D10	831.0	637.3	76.69
D11	280.2	126.5	45.15
D12	524.1	168.2	32.09
D13	206.5	111.6	54.04
D14	335.4	205.4	61.24
D15	203.5	112.5	55.28

Average Coefficient of Variation = 52.69

ANALYSIS OF DEMAND SERIES

TABLE 6-1

CHAPTER 7

SUMMARY

Centering on manufacturers faced with a replacement market subject to independent demand, this thesis evaluated alternative forecasting methods. Since an understanding of a forecasting technique was considered a prerequisite to its acceptance, several simple forecasting models were explored. These basic techniques were extended by the addition of monitoring systems. The seven models examined were the six-month moving average, six-month weighted moving average, single exponential smoothing with tracking, single exponential smoothing with a linear trend plus tracking, double exponential smoothing with tracking, single exponential smoothing with adaptive control, and double exponential smoothing with adaptive control.

With this foundation, criteria for the evaluation of model goodness-of-fit were developed. Next, the model parameters of the non-adaptive exponential smoothing models were optimized. Finally, all models were compared on the basis of the evaluation criteria. As a result of this analysis, the six-month weighted moving average model was selected as the best forecasting technique.

CHAPTER 8

RECOMENDATIONS FOR FUTURE WORK

Future work should include the examination of more data and further extensions of model innovations explored in this thesis.

The expolitory nature of this thesis precludes any firm conclusions concerning the appropriate model choice given a replacement market with independent demand. Additional data must be examined before the results of this analysis can be generalized.

To extend this study, the adaptive control tracking system could be amended. These preliminary results highlight a tendency for more responsive models to become excessively reactive. Altering the modification of the Alpha parameter proposes one possible solution to this problem. Specifically, by discounting the modified Alpha, the system would become less reactive.

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APPENDIX

Analysis of Forecasting Models

Analysis of Forecasting Models

Data Set D1

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	23.81	0.6146	2	1
WMA	22.91	0.6692	1	4
SET	24.79	0.6478	5	2
SLT	24.09	0.6935	4	5
DET	23.98	0.6610	3	3
SAC	28.52	0.8805	7	7
DAC	27.96	0.8663	6	6

Table A-1

Analysis of Forecasting Models

Data Set D2

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	51.68	1.1278	7	7
WMA	47.25	1.0345	2	2
SET	50.48	0.9855	4	1
SLT	51.24	1.0386	6	3
DET	50.53	1.0549	5	4
SAC	47.87	1.0862	3	6
DAC	46.96	1.0787	1	5

Table A-2

Analysis of Forecasting Models

Data Set D3

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	31.88	0.7312	6	1
WMA	32.52	0.7839	7	3
SET	30.10	0.7415	4	2
SLT	30.33	0.8414	5	5
DET	29.96	0.7866	3	4
SAC	29.51	0.9385	2	6
DAC	29.45	0.9649	1	7

Table A-3

Analysis of Forecasting Models

Data Set D4

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	40.06	0.6753	1	1
WMA	42.47	0.7779	2	5
SET	57.35	0.6970	5	2
SLT	57.05	0.7128	4	4
DET	55.54	0.7042	3	3
SAC	75.69	1.1196	7	7
DAC	71.16	1.1058	6	6

Table A-4

Analysis of Forecasting Models

Data Set D5

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	40.06	0.8694	7	6
WMA	39.05	0.9967	6	7
SET	36.25	0.9106	2	3
SLT	36.64	0.8910	3	1
DET	35.52	0.9154	1	4
SAC	38.67	0.9209	5	5
DAC	38.01	0.8990	4	2

Table A-5

Analysis of Forecasting Models

Data Set D6

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	26.56	0.8285	2	3
WMA	15.90	0.7871	1	1
SET	26.81	0.8302	5	2
SLT	40.81	1.1389	7	7
DET	38.56	0.9850	6	6
SAC	33.95	0.9109	3	4
DAC	34.47	0.9454	4	5

Table A-6

Analysis of Foercasting Models

Data Set D7

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	62.14	0.7709	6	2
WMA	53.93	0.7268	1	1
SET	54.42	0.9697	2	5
SLT	65.26	1.0489	7	7
DET	58.07	1.0066	4	6
SAC	59.15	0.9241	5	4
DAC	58.04	0.9181	3	3

Table A-7

Analysis of Forecasting Models

Data Set D8

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	47.23	0.3925	2	2
WMA	43.70	0.4501	1	5
SET	65.57	0.4022	7	3
SLT	62.33	0.4132	5	4
DET	62.84	0.3924	6	1
SAC	57.15	0.7315	3	7
DAC	55.98	0.7199	4	6

Table A-8

Analysis of Forecasting Models

Data Set D9

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	98.08	1.1428	5	4
WMA	83.93	0.9586	4	2
SET	69.01	1.4936	1	7
SLT	77.53	1.4271	3	5
DET	73.43	1.4445	2	6
SAC	126.25	0.9376	7	1
DAC	106.87	1.0628	6	3

Table A-9

Analysis of Forecasting Models

Data Set D10

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	82.37	0.9263	3	6
WMA	89.43	0.9581	7	7
SET	86.04	0.6862	6	3
SLT	78.57	0.7359	1	5
DET	80.26	0.7041	2	4
SAC	85.39	0.6720	5	1
DAC	83.91	0.6728	4	2

Table A-10

Analysis of Forecasting Models

Data Set D11

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	43.83	0.8264	5	5
WMA	42.45	0.8264	3	6
SET	38.87	0.8455	1	7
SLT	44.34	0.7789	6	1
DET	40.92	0.8026	2	2
SAC	35.32	0.8144	7	3
DAC	43.82	0.8188	4	4

Table A-11

Analysis of Forecasting Models

Data Set D12

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	29.78	0.9512	2	6
WMA	27.76	0.8788	1	2
SET	31.41	0.8754	3	1
SLT	32.71	0.9969	5	7
DET	31.66	0.9377	4	3
SAC	37.96	0.9492	7	5
DAC	37.26	0.9405	6	4

Table A-12

Analysis of Forecasting Models

Data Set D13

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	54.14	0.5059	3	1
WMA	57.00	0.5445	6	2
SET	54.15	0.5702	4	5
SLT	53.11	0.6253	2	7
DET	52.37	0.6039	1	6
SAC	58.44	0.5545	7	4
DAC	56.47	0.5535	5	3

Table A-13

Analysis of Forecasting Models

Data Set D14

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	56.72	0.6103	7	1
WMA	55.81	0.7260	6	6
SET	50.89	0.6978	4	3
SLT	51.16	0.6986	5	5
DET	50.71	0.6985	3	4
SAC	50.53	0.6932	2	2
DAC	48.47	0.7328	1	7

Table A-14

Analysis of Forecasting Models

Data Set D15

Forecasting Model	AAPE	Theil's U-Statistic	Rank Using AAPE	Rank Using Theil's U-Statistic
MAV	43.73	0.9783	5	3
WMA	42.21	0.8529	3	1
SET	38.94	1.0104	1	5
SLT	44.07	1.0922	6	7
DET	39.98	1.0860	2	6
SAC	45.46	0.9560	7	2
DAC	43.47	1.0011	4	4

Table A-15

VITA

PERSONAL HISTORY

Name:	James M. Confer
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Springfield North High School — Graduated 1975
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